Homework 4

Number Theory and Cryptography (201912400327)

Due Date: June 10, 2024

Question 1.

Show the identities:

- $[a_0, \cdots, a_n] = a_0 + \frac{1}{[a_1, \cdots, a_n]}$
- $[a_0, \cdots, a_n] = [a_0, \cdots, a_{n-1} + \frac{1}{a_n}]]$
- $[a_0, \cdots, a_n] = [a_0, \cdots, a_n - 1, 1]$
- $[a_0, \cdots, a_n]^{-1} = [0, a_0, \cdots, a_n]$

Question 2.

Compute the continued fraction expansions of $\sqrt{5}$ and the golden ratio $\phi = \frac{1+\sqrt{5}}{2}$.

Question 3.

Which quadratic irrational has the continued fraction expansion $[1, \overline{6, 2}\,]$?

Question 4.

Show that every nonzero rational number can be represented in exactly two ways by a finite simple continued fraction.

Question 5.

Verify that

$$\sqrt{61} = [7, \overline{1, 4, 3, 1, 2, 2, 1, 3, 4, 1, 14}]$$

and solve Pell's equation $x^2 - 61y^2 = 1$.