Homework 4

Number Theory and Cryptography (201912400327)

Due Date: June 10, 2024

Question 1.

Evaluate the following Legendre symbols:

- $\left(\frac{85}{101}\right)$
- $\left(\frac{29}{541}\right)$
- $\left(\frac{101}{1987}\right)$

Question 2.

Determine whether $x^2 \equiv 150 \pmod{1009}$ is solvable.

Question 3.

Use quadratic reciprocity to evaluate $\left(\frac{7}{p}\right)$ based on the residue class of $p \pmod{28}$.

Question 4.

Prove that if $p \mid (n^2 - 5)$ for some integer n, then $p \equiv 1$ or $4 \pmod{5}$.

Question 5.

- Characterize all primes p such that $\left(\frac{10}{p}\right) = 1$.
- Characterize all primes p such that $\left(\frac{5}{p}\right) = -1$