Homework 2

Number Theory and Cryptography (201912400327)

Due Date: May 27, 2024

Question 1.

Find the remainder when 98! is divided by 101.

Question 2.

Compute

- $15^{10235} \pmod 7$
- $120^{13} \pmod{11}$
- $3^{2023} \pmod 7$
- $3^{-1} \pmod{28}$
- $5^{12345678} \pmod{11}$.

Question 3.

Solve the following system of linear congruences:

$$\begin{aligned} x &\equiv 2 \pmod 3 \\ x &\equiv 4 \pmod 5 \\ x &\equiv 3 \pmod 7 \end{aligned}$$

Question 4.

Let ϕ be the Euler phi function. For what values of n is $\phi(n)$ even?

Question 5.

Explicitly find a primitive root modulo 49.

Question 6.

Is the number $n = 3^{2024} - 2024$ prime? Show your work.